

Academic Scholarship 2012

Preliminary Examination

## MATHEMATICS

## Time Allowed: $11 / 2$ hours

- Calculators may NOT be used.
- Write your answers on lined paper and show as much working as possible. Answers without clear logical working will gain little credit.
- Do not spend too long on any single question. If you are having difficulty with a particular question, move on and return to it at the end if you have time. Do not be concerned if you cannot answer all of the questions.
- At the end of the examination, hand in both the question paper and your answers with your name clearly indicated on all sheets.

1. Work out
(a) $40 \%$ of 350 .
(b) $240 \times 25$
(c) $5^{3}$
(d) $\sqrt{64}$
(e) $3 \frac{1}{2} \times 1 \frac{1}{3}$
(f) $\frac{\frac{3}{5}-\frac{1}{4}}{\frac{7}{8} \div \frac{1}{4}}$
(g) $\frac{2}{3} \times 12+3 \div 0.2$
2. (a) Remove brackets :

$$
2(3-4 x)
$$

(b) Factorise fully :

$$
6 x y^{2}-12 x^{2} y
$$

(c) Multiply out the brackets and simplify :

$$
(x+3)(x+2)
$$

(d) If $a=2, b=-3$ and $c=\frac{1}{4}$, work out the value of:
(i) $a^{2}-b^{2}$
(ii) $\frac{a}{c}-b$
(iii) $-\frac{2}{3} a b c$
3. Solve for $x$ :
(a) $5 x-3=9-x$
(b) $\frac{2}{3}(2 x-3)=8$
4. $\quad$ Solve for $x$ and $y: \quad 2 x-3 y=13$

$$
3 x+2 y=0
$$

5. A rectangle has adjacent sides of length $(2 x+3) \mathrm{cm}$ and $(5 x-1) \mathrm{cm}$. The perimeter is 102 cm . Find the value of $x$.
6. The best sprinters in the world run the 100 metres in about 10 seconds.
(a) Work out their average speed during the race in kilometres per hour (km/h).
(b) Using a simple conversion (of kilometres to miles), work out their approximate average speed in miles per hour (mph).
7. A class of 8 pupils sat an Algebra test. Their average mark was $65 \%$. Jenny was absent for the test, but sat it the following day. When her mark was included, the class average was $60 \%$. What was her mark for the test ?
8. 



Diagram not accurately drawn.

The diagram shows three squares of the same size. What is the size (in degrees) of the angle marked $x$ ?
9. James is a property developer. He bought two homes and sold them both a year later. He sold each of them for $£ 990000$. On the first home, he made a profit of $10 \%$, but on the second home he made a loss of $10 \%$. Overall, what percentage profit or loss has he made on the two houses, or was his profit/loss exactly zero?
10. When a barrel is $30 \%$ empty, it contains 30 litres more than when it is $30 \%$ full. How many litres does the barrel hold when it is full?
11. (a) Three different Mathematics books are to be placed side by side on a shelf, as shown below :


Let's call the books A, B and C. Using these letters, write down all the different ways in which the books can be placed.
(b) I now add a fourth Mathematics book (call it D) to the collection. Work out the number of different ways in which the four books can be arranged on the shelf.
(note : you will not get full marks by just writing out all the different ways and counting them - try to think of a clever method of calculating the answer, and remember to show your working so that your method is clear).
(c) Six different Mathematics books are to be placed on a shelf, as shown below.

(i) In how many different ways can this be done ?
(ii) I now decide that two particular books must occupy the two 'end' positions (marked with a * below). (note : it does not matter which goes on which end).


In how many different ways can I now arrange the books?
12. At a holiday camp, the ratio of boys to girls is $3: 4$. The ratio of girls to adults is $5: 7$. What is the ratio of children to adults at the camp ?
13.


In the diagram, triangle $X Y Z$ is isosceles, with $X Y=X Z$.
Three angles have been labelled as $r, p$ and $q$.
Find the size of $r$ in terms of $p$ and $q$.
14. Find the value of $\frac{1}{x+2}$ if $\frac{1}{x}=3.5$
15. In Mathematics, $n$ ! is called $n$ factorial and this is how it works :

$$
n!=1 \times 2 \times 3 \times 4 \times \ldots \ldots . . \times n
$$

So, for example :
$3!=1 \times 2 \times 3$ and $7!=1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7$
Amazingly, the exact number of seconds in 6 weeks is equal to $b!$ ( $b$ is a whole number).

Work out the value of $b$.

E N D

